Liabilities and Bonds (Time Value of Money)

Hurray! You've just won a million dollars in the lottery! However, your prize can only be claimed in one of two ways. The first, is to claim the million dollars today, and the second, is to claim the million dollars in one years' time. As any good financial accounting student would do, you decide to claim the money today. Why is it better to receive one million dollars today?

There could be several reasons, but consider the following: suppose you took the one million dollars you received today and put it in a bank account that earns 1% annual interest. When you come back to the bank a year from now, you will have one million and ten thousand dollars in your account. Your bank account is now worth 101% of its original amount. Had you decided to claim your prize in a year's time, you wouldn't have made that extra ten thousand dollars. Because the money you have on hand today has this potential for growth, we always treat it as if it will be worth more than its face value. Thus, one million dollars today is always worth more than one million dollars in one year, or any future date.

This idea is called the "Time Value of Money", where we treat money as if its **value decreases over time**. This is because that money can earn interest over time.

We refer to the value of an amount of money today as its "Present Value", and the value of that same amount in the future as its "Future Value." It is important to know how to convert between present value and future value when accounting for **financial assets that mature over time** like bonds and stocks. The formula to convert future value to present value is the following: *(see video for formula)*

Where **PV** is present value, **FV** is future value, **i** is the annual interest rate, and **n** is the number of periods. Going from future value to present value is also known as **discounting,** because we are going from an amount plus its future interest, back down to what the amount is worth today **without its interest**.

To go from present value to future value, we use the following formula: *(see video for formula)*

Going from present value to future value is also known as **compounding**, because we are going from an amount today without interest, up to the amount **plus its interest in the future**.

Let's revisit our million dollar lottery ticket. If you decide to get your prize money now, the present value of that money would be one million dollars. However, you know now that the million dollars won't be worth one million dollars in a year's time. We can use the **future value formula** to calculate the **future value** of your one million dollars. Let's assume the current interest rate is four percent, and see how much interest we accumulate in a year: *(see video for formula)*

Wow, it looks like choosing to receive the money now was the right move, because you can get an additional forty thousand dollars' worth of accrued interest! You're excited to tell your family about your new fortune, so you call your grandma first. She is absolutely stunned at the news, but now she has a question for you. She's wondering how much she should have invested sixty years ago to have a million dollars today, the equivalent of your prize money. Can you figure that out?

Remember that we also have a formula to adjust a future value back to its present value. In this situation, the one million dollars is the future value, and we want to discount it back to the value it would've been 60 years ago. Let's assume a 4% interest rate again and see the discount in action!

Hmm, that doesn't seem like a whole lot. Apparently your one million dollars today was only worth $95,060.40 sixty years ago!

We just learned how to adjust between present and future values for a single amount. But what if the transaction we are interested in involves a series of payments? For example, what if the lottery prize of $1M was to be paid to you in 5 equal sums of $200,000 instead?

We refer to a series of equal dollar amounts to be received or paid periodically as an **annuity.** Examples of annuities include loan agreements, payments made in installments, mortgage notes, lease or rental contracts, and pension plans.

There are two kinds of annuities that we deal with: **ordinary annuities, and annuities due**. **Ordinary annuities** are amounts that are paid at the **end** of each period, while **annuities due** are paid at the **beginning** of each period.

Let's begin with the ordinary annuity. How would we find its present value? Remember that an annuity is simply a series of single amount payments that you receive at regular intervals. For our million dollars, we are getting a single amount of $200,000 one year from now, then two years from now, then three years from now, all the way to five years from now. That means we can use the present value formula we already know to derive the value of an annuity, because the present value of the annuity will simply be the present value of all the payments from the annuity.

In this case, the present value of the annuity is the present value of $200,000 compounded at 4% for one year, plus the present value of $200,000 compounded at 4% for two years, plus the present value of $200,000 compounded at 4% for three years, all the way to five years. A similar line of thinking will get you the future value of an annuity - simply sum up the future values of the individual payments instead.

However, that's a lot of work, especially if you have a lot of compounding periods. Try doing that for a 20 year mortgage! If you're wondering whether there's a formula for that, there is. For the ordinary annuity, we use the following formula for present value: *(see video for formula)*

And this one, for future value: *(see video for formula)*

Where PV is present value, **FV** is future value, **A** is annuity payment per period, **i** is interest rate, and **n** is the number of periods the annuity is paid for.

If we are interested in the present and future values of the **annuity due**, we need to multiply the previous formulas by $(\mathbf{1} + \mathbf{i})$. This will account for the extra period that is at the very beginning, as we start accumulating interest at the beginning of each period now. Thus, the **present value** is: *(see video for formula)*

And the **future value** is: *(see video for formula)*

Back to your lottery prize money. Let's say there was a third option, where you can choose to receive $250,000 once a year, for four years. At the same interest rate of 4%, what's the future value of this annuity?

In this scenario, the annuity is $250,000, the interest is 4%, and the number of periods is 4. With this, you will end up receiving $1,061,616 (one million, sixty one thousand and six Hundred and sixteen dollars) after receiving all your payments at the end of the four-year annuity payout period.

To calculate exactly how much our money would be worth in the future, we use the **future value** or **compounding** formula, because interest is compounded or added on to our initial amount. We can use the **present value** or **discounting** formula to go in the other direction, to see what amount we would need today to arrive at a certain amount in the future. We also looked at the annuity, where we deal with equal amounts of money being paid out over multiple periods. Present and future values of annuities are made up of the sum of the present or future values of the individual payments. Because of this, annuities usually differ from lump sums, even if their total amounts appear to be the same amount on the surface! Always remember to keep the time value of money in mind the next time you are dealing with any amount that matures over time.